CEGE046 / GEOG3051
Principles & Practice of Remote Sensing (PPRS)
2: Radiation (i)

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Outline: lecture 2 & 3

• Core principles of electromagnetic radiation (EMR)
  – solar radiation
  – blackbody concept and radiation laws

• EMR and remote sensing
  – wave and particle models of radiation
  – regions of EM spectrum
  – radiation geometry, terms, units
  – interaction with atmosphere
  – interaction with surface

• Measurement of radiation
Conceptual basis for understanding EMR
Terms, units, definitions
Provide basis for understanding type of information that can be (usefully) retrieved via Earth observation (EO)
Why we choose given regions of the EM spectrum in which to make measurements
Remote sensing process: recap

• Note various paths
  – Source to sensor direct?
  – Source to surface to sensor
  – Sensor can also be source
    • RADAR, LiDAR, SONAR
    • i.e. “active” remote sensing

• Reflected and emitted components
  – What do these mean?

• Several components of final signal captured at sensor
Energy transport

• Conduction
  – transfer of molecular kinetic (motion) energy due to contact
  – heat energy moves from $T_1$ to $T_2$ where $T_1 > T_2$

• Convection
  – movement of hot material from one place to another
  – e.g. Hot air rises

• Radiation
  – results \textit{whenever an electrical charge is accelerated}
  – propagates via EM waves, through vacuum & over long distances hence of interest for remote sensing
Electromagnetic radiation: wave model

- James Clerk Maxwell (1831-1879)

**Wave model of EM energy**

- Unified theories of electricity and magnetism (via Newton, Faraday, Kelvin, Ampère etc.)
- Oscillating electric charge produces magnetic field (and vice versa)
- Can be described by 4 simple (ish) differential equations
- Calculated speed of EM wave in a vacuum
Electromagnetic radiation

• EM wave is

• Electric field (E) perpendicular to magnetic field (M)

• Travels at velocity, c \((3 \times 10^8 \text{ ms}^{-1}, \text{ in a vacuum})\)
Wave: terms

• All waves characterised by:
  • Wavelength, $\lambda$ (m)
  • Amplitude, $a$ (m)
  • Velocity, $v$ (m/s)
  • Frequency, $f$ (s$^{-1}$ or Hz)
  • Sometimes period, $T$ (time for one oscillation i.e. 1/f)
Wave: terms

• Velocity, frequency and wavelength related by

\[ \lambda \propto \frac{1}{f} \]

• \( f \) proportional to \( 1/\lambda \) (constant of proportionality is wave velocity, \( v \) i.e.)

\[ v = f\lambda \]
• Note angles in **radians** (rad)

  • $360^\circ = 2\pi$ rad, so 1 rad = $360/2\pi = 57.3^\circ$

  • Rad to deg. ($\ast 180/\pi$) and deg. to rad ($\ast \pi/180$)
Maxwell’s Equations

• 4 simple (ish) equations relating vector electric ($\mathbf{E}$) and vector magnetic fields ($\mathbf{B}$)

• $\varepsilon_0$ is permittivity of free space

• $\mu_0$ is permeability of free space

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \\

\nabla \cdot \mathbf{B} = 0 \\

\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\

\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \\

\varepsilon_0 = \frac{1}{c^2 \mu_0}
\]
Maxwell’s Equations

1. Gauss’ law for electricity: the electric flux out of any closed surface is proportional to the total charge enclosed within the surface.

\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \]

2. Gauss’ law for magnetism: the net magnetic flux out of any closed surface is zero (i.e. magnetic monopoles do not exist).

\[ \nabla \cdot B = 0 \]

3. Faraday’s Law of Induction: line integral of electric field around a closed loop is equal to negative of rate of change of magnetic flux through area enclosed by the loop.

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]

4. Ampere’s Law: for a static electric field, the line integral of the magnetic field around a closed loop is proportional to the electric current flowing through the loop.

\[ \nabla \times B = \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \]

Note: \( \nabla \cdot \) is ‘divergence’ operator and \( \nabla \times \) is ‘curl’ operator

http://en.wikipedia.org/wiki/Maxwell's_equations
EM Spectrum

• EM Spectrum
  • Continuous range of EM radiation
  • From very short wavelengths (<300x10^{-9}m)
    • high energy
  • To very long wavelengths (cm, m, km)
    • low energy
  • Energy is related to wavelength (and hence frequency)
Units

• EM wavelength $\lambda$ is m, but various prefixes
  • cm ($10^{-2}$m)
  • mm ($10^{-3}$m)
  • micron or micrometer, $\mu$m ($10^{-6}$m)
  • Angstrom, Å ($10^{-8}$m, used by astronomers mainly)
  • nanometer, nm ($10^{-9}$)

• $f$ is waves/second or Hertz (Hz)

• NB can also use wavenumber, $k = 1/\lambda$ i.e. m$^{-1}$
• Energy radiated from sun (or active sensor)
• Energy $\propto 1/$wavelength $(1/\lambda)$
  – shorter $\lambda$ (higher f) == higher energy
  – longer $\lambda$ (lower f) == lower energy
from http://rst.gsfc.nasa.gov/Intro/Part2_4.html
• We will see how energy is related to frequency, \( f \) (and hence inversely proportional to wavelength, \( \lambda \))

• When radiation passes from one medium to another, speed of light \( c \) and \( \lambda \) change, hence \( f \) stays the same
Electromagnetic spectrum: visible

- Visible part - very small part
  - from visible blue (shorter $\lambda$)
  - to visible red (longer $\lambda$)
  - $\approx 0.4$ to $\approx 0.7 \mu m$

Violet: 0.4 - 0.446 $\mu m$
Blue: 0.446 - 0.500 $\mu m$
Green: 0.500 - 0.578 $\mu m$
Yellow: 0.578 - 0.592 $\mu m$
Orange: 0.592 - 0.620 $\mu m$
Red: 0.620 - 0.7 $\mu m$
Electromagnetic spectrum: IR

- Longer wavelengths (sub-mm)
- Lower energy than visible
- Arbitrary cutoff
- IR regions covers
  - reflective (shortwave IR, SWIR)
  - and emissive (longwave or thermal IR, TIR)
  - region just longer than visible known as near-IR, NIR.
Electromagnetic spectrum: microwave

- Longer wavelength again
  - RADAR
  - mm to cm
  - various bands used by RADAR instruments
  - long $\lambda$ so low energy, hence need to use own energy source (active $\mu$wave)
Blackbody

• All objects above absolute zero (0 K or -273° C) radiate EM energy (due to vibration of atoms)

• We can use concept of a perfect blackbody
  • Absorbs and re-radiates all radiation incident upon it at maximum possible rate per unit area (Wm⁻²), at each wavelength, λ, for a given temperature T (in K)

• Energy from a blackbody?
Stefan-Boltzmann Law

• Total emitted radiation from a blackbody, $M_\lambda$, in Wm$^{-2}$, described by Stefan-Boltzmann Law

\[ M_\lambda = \sigma T^4 \]

• Where $T$ is temperature of the object in K; and $\sigma = \text{Stefan-Boltzmann constant} = 5.6697 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

• So energy $\propto T^4$ and as $T \uparrow$ so does $M$
  
  • $T_{\text{sun}} \approx 6000\text{K} \ M_{\lambda, \text{sun}} \approx 73.5 \text{ MWm}^{-2}$
  
  • $T_{\text{Earth}} \approx 300\text{K} \ M_{\lambda, \text{Earth}} \approx 460 \text{ Wm}^{-2}$
Stefan-Boltzmann Law

![Graph showing energy per unit wavelength (Wm⁻²) against wavelength (µm) for Sun (T = 6000K) and Earth (T = 300K).]
Stefan-Boltzmann Law

• Note that peak of sun’s energy around 0.5 µm
  • negligible after 4-6µm
• Peak of Earth’s radiant energy around 10 µm
  • negligible before ~ 4µm
• Total energy in each case is area under curve
Stefan-Boltzmann Law

• Generalisation of Stefan-Boltzmann Law
  • radiation $\Phi$ emitted from unit area of any plane surface with emissivity of $\varepsilon$ (<1) can be written
  • $\Phi = \varepsilon \sigma T^n$ where $n$ is a numerical index
  • For ‘grey’ surface where $\varepsilon$ is nearly independent of $\lambda$, $n = 4$
  • When radiation emitted predominantly at $\lambda < \lambda_m$, $n > 4$
  • When radiation emitted predominantly at $\lambda > \lambda_m$, $n < 4$
Peak $\lambda$ of emitted radiation: Wien’s Law

• Wien deduced from thermodynamic principles that energy per unit wavelength $E(\lambda)$ is function of $T$ and $\lambda$

$$E(\lambda) = \frac{f(\lambda T)}{\lambda^5}$$

• At what $\lambda_m$ is maximum radiant energy emitted?

• Comparing blackbodies at different $T$, note $\lambda_m T$ is constant, $k = 2897\mu\text{mK}$ i.e. $\lambda_m = k/T$

• $\lambda_{m,\text{sun}} = 0.48\mu\text{m}$

• $\lambda_{m,\text{Earth}} = 9.66\mu\text{m}$
• AKA Wien’s Displacement Law
• Increase (displacement) in $\lambda_n$ as $T$ reduces
• Straight line in log-log space
Particle model of radiation

• Hooke (1668) proposed wave theory of light propagation (EMR) (Huygens, Euler, Young, Fresnel…)

• Newton (~1700) proposed corpuscular theory of light (after al-Haytham, Avicenna ~11th C, Gassendi ~ early 17th C)
  • observation of light separating into spectrum

• Einstein explained photoelectric effect by proposing photon theory of light
  • Photons: individual packets (quanta) of energy possessing energy and momentum

• Light has both wave- and particle-like properties
  • Wave-particle duality
Particle model of radiation

• EMR intimately related to atomic structure and energy

• Atom: +ve charged nucleus (protons + neutrons) & -ve charged electrons bound in orbits

  • Electron orbits are fixed at certain levels, each level corresponding to a particular electron energy

  • Change of orbit either requires energy (work done), or releases energy

  • Minimum energy required to move electron up a full energy level (can’t have shift of 1/2 an energy level)

  • Once shifted to higher energy state, atom is *excited*, and possesses potential energy

  • Released as electron falls back to lower energy level
Particle model of radiation

• As electron falls back, quantum of EMR (photons) emitted
  • electron energy levels are unevenly spaced and characteristic of a particular element (basis of spectroscopy)

• Bohr and Planck recognised discrete nature of transitions

• Relationship between frequency of radiation (wave theory) of emitted photon (particle theory)

\[ E = hf \]

• E is energy of a quantum in Joules (J); h is Planck constant \( (6.626 \times 10^{-34} \text{Js}) \) and f is frequency of radiation
• If we remember that velocity \( v = f\lambda \) and in this case \( v \) is actually \( c \), speed of light then

\[
E = \frac{hc}{\lambda}
\]

• Energy of emitted radiation is inversely proportional to \( \lambda \)
  • longer (larger) \( \lambda \) == lower energy
  • shorter (smaller) \( \lambda \) == higher energy

• Implication for remote sensing: harder to detect longer \( \lambda \) radiation (thermal for e.g.) as it has lower energy
Particle model of radiation

From: http://abyss.uoregon.edu/~js/glossary/bohr_atom.html
### Particle model of radiation: atomic shells

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<tr>
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http://www.tmeg.com/esp/e_orbit/orbit.htm
Planck’s Law of blackbody radiation

• Planck was able to explain energy spectrum of blackbody
• Based on quantum theory rather than classical mechanics

\[ E(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{\frac{hc}{e^{\frac{hc}{\lambda kT}} - 1}} \]

• \( \frac{dE(\lambda)}{d\lambda} \) gives constant of Wien’s Law
• \( \int E(\lambda) \) over all \( \lambda \) results in Stefan-Boltzmann relation
• Blackbody energy function of \( \lambda \), and \( T \)

http://www.tmeg.com/esp/e_orbit/orbit.htm
Planck’s Law

- Explains/predicts shape of blackbody curve
- Use to predict how much energy lies between given $\lambda$
  - Crucial for remote sensing

$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$
Consequences of Planck’s Law: plants

• Chlorophyll a,b absorption spectra

• Photosynthetic pigments
  • Driver of (nearly) all life on Earth!
  • Source of all fossil fuel
Consequences of Planck’s Law: us

Cones: selective sensitivity

Rods: monochromatic sensitivity

http://www.photo.net/photo/edscott/vis00010.htm
Applications of Planck’s Law

• Fractional energy from 0 to \( \lambda \) i.e. \( F_{0 \rightarrow \lambda} \)? Integrate Planck function

• Note \( E_{b\lambda}(\lambda, T) \), emissive power of bbody at \( \lambda \), is function of product \( \lambda T \) only, so....

\[
F_{0 \rightarrow \lambda}(\lambda, T) = \frac{E_{0 \rightarrow \lambda}(\lambda, T)}{\sigma T^4} = \int_0^{\lambda T} d(\lambda, T) \frac{E_{b\lambda}(\lambda, T)}{\sigma T^5}
\]

Radiant energy from 0 to \( \lambda \)

Total radiant energy
for \( \lambda = 0 \) to \( \lambda = \infty \)
Applications of Planck’s Law: example

• Q: what fraction of the total power radiated by a black body at 5770 K fall, in the UV ($0 < \lambda \leq 0.38 \mu m$)?

• Need table of integral values of $F_{0 \rightarrow \lambda}$

• So, $\lambda T = 0.38 \mu m \times 5770 K = 2193 \mu m K$

• Or 2.193x10³ $\mu m K$ i.e. between 2 and 3

• Interpolate between $F_{0 \rightarrow \lambda}$ (2x10³) and $F_{0 \rightarrow \lambda}$ (3x10³)

\[
\frac{F_{0 \rightarrow 0.38} (\lambda, T) - F_{0 \rightarrow 0.38} (2 \times 10^3)}{F_{0 \rightarrow 0.38} (3 \times 10^3) - F_{0 \rightarrow 0.38} (2 \times 10^3)} = \frac{2.193 - 2}{3 - 2} = 0.193
\]

\[
\frac{F_{0 \rightarrow 0.38} (\lambda, T) - 0.067}{0.273 - 0.067} = 0.193
\]

• Finally, $F_{0 \rightarrow 0.38} = 0.193 \times (0.273 - 0.067) + 0.067 = 0.11$

• i.e. ~11% of total solar energy lies in UV between 0 and 0.38 $\mu m$
Applications of Planck’s Law: exercise

Show that ~38% of total energy radiated by the sun lies in the visible region (0.38µm < \lambda \leq 0.7µm) assuming that solar T = 5770K

Hint: we already know F(0.38µm), so calculate F(0.7µm) and interpolate

<table>
<thead>
<tr>
<th>\lambda T (\mu mK x 10^3)</th>
<th>F_{0 \rightarrow \lambda}(\lambda T) (dimensionless)</th>
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Departure from BB assumption?
Recap

• Objects can be approximated as blackbodies
  • Radiant energy $\propto T^4$

• EM spectrum from sun a continuum peaking at $\sim 0.48 \mu m$
  • $\sim 39\%$ energy between 0.38 and 0.7 in visible region

• Planck’s Law - shape of power spectrum for given $T$ (Wm$^{-2}$ $\mu m^{-1}$)
  • Integrate over all $\lambda$ to get total radiant power emitted by BB per unit area
    • Stefan-Boltzmann Law $M = \sigma T^4$ (Wm$^{-2}$)
  • Differentiate to get Wien’s law
    • Location of $\lambda_{\text{max}} = k/T$ where $k = 2898 \mu mK$