Radiative Transfer Theory at Optical wavelengths applied to vegetation canopies: part 1: Introduction and Canopy Elements

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1. Introduction

1.1 Aim and Scope of these Notes

The purpose of these notes is to introduce concepts of the radiative transfer approach to modelling scattering of electromagnetic radiation by vegetation canopies and to review alternative approaches to modelling. Whilst we concentrate on application to optical wavelengths here, the theory is also appropriate to considerations in the thermal and microwave regimes. Since the underlying theory was first formulated for stellar atmospheres (Chandrasekhar, 1960) the theory is of course also applicable to modelling of atmospheric scattering.

Fung (1994) presents a range of motivations for the development of theoretical models of scattering such as those presented here. These can be stated as:

1. to assist data interpretation by permitting the calculation of the remote sensing (RS) signal as a function of fundamental biophysical variables through a consideration of the physics of scattering;
2. to permit studies of signal sensitivity to biophysical or system parameters;
3. to provide a tool for the interpolation or extrapolation of data;
4. to provide ‘forward model’ simulations for deriving estimates of biophysical parameters through model inversion;
5. to aid in experimental design.

The notes are aimed at MSc students taking the University of London MSc in Remote Sensing. We have primary educational aims here of enabling students to gain an understanding of the theory and its applications and permitting students access to the vast literature in this and related areas. We aim the notes at non mathematical experts from a wide variety of disciplines who wish to develop or (more typically) use models for remote sensing applications, although some understanding of vectors, matrices, dot products, basic calculus and complex numbers is required. For more detail and options within the theory, readers are referred in particular to the following references: Sobolev (1975), Ross (1981), Myneni et al. (1989), Ulaby and Elachi (1990), and Fung (1994).

As a starting point, we assume students have at least a basic understanding of remote sensing and physical concepts in radiation such as polarised waves. A useful introduction to polarimetry is provided by Ulaby and Elachi (1990) (Chapter 1). Concepts in radiometry are usefully dealt with by Slater (1980) (Chapters 3-5).

1.2 Applicability of the Theory

The radiative transfer (RT) approach (otherwise known as transport theory) is a heuristic treatment of multiple scattering of radiation which assumes that there is no correlation between fields considered and so that the addition of power terms, rather than the addition of fields, is appropriate (Ulaby and Elachi, 1990). Although diffraction and interference effects can be included in consideration of scattering from and absorption by single particles, RT theory does not consider diffraction effects (ibid., p. 134). A more accurate, but difficult to formulate, approach is to start with a consideration of basic differential equations such as Maxwell’s equations (ibid.; chapter 1; Slater, 1980; p. 55).

We will develop here the radiative transfer equation for the case of a plane parallel medium (of air) embedded with infinitessimal oriented scattering objects at low density (leaves, stems etc., and an underlying soil or other dense medium) ‘suspended’ in air (a ‘turbid medium’). We consider only absorption and scattering events (i.e. no emission). We consider the canopy to be of horizontally infinite but vertically finite extent filled with scattering elements defined continuously over the canopy space (no explicit gaps which are correlated between any canopy layers). We will further assume the canopy to be horizontally homogeneous (i.e. scatterer density is constant over the horizontal extent of the canopy) although this is not a strict requirement of the theory (see Myneni et al., 1989; p.6). We will also deal only with a random (Poisson) distribution of vegetation in detail in these notes. The reader is referred to Myneni et al. (1989; p. 8) for consideration of other spatial distributions. Considering only low density canopies (1% or less by volume) of small scatterers aids the applicability of the no field correlation assumption (scatterers are assumed to be in the ‘far field’ of one another). This assumption means that the theory presented is not directly applicable for dense media such as snow or sea ice (see Fung, 1994; p. 373 on how to approach this problem). Assuming the canopy to be in air allows for power absorption by the surrounding medium to be ignored (Ulaby and Elachi, 1990; p. 136). Assuming horizontal homogeneity allows us to deal with radiation transport in
only one dimension (a ‘1-D solution’), although the theory can be applied to 3-D scattering problems. Consideration of a medium containing oriented scatterers (developed for optical wavelengths by Ross, 1981) is appropriate for vegetation canopies and provides a point of departure from consideration of atmospheric scattering. See Ulaby and Elachi (1990) pp. 185-186 for a further discussion of the deficiencies of the radiative transfer approach. Later in the course, we will review alternative approaches to modelling which overcome some of these issues.

We can develop both scalar and vector forms of the RT equation. In the microwave domain, waves are not unpolarised, and we must consider polarisation by using the vector form. At optical wavelengths, the scalar form is generally used, although the vector form is also used as the basis for many atmospheric examples and e.g. for considering aspects of specular effects from soils and leaves. The vector form provides four coupled integro-differential equations, whereas a single equation is used in the scalar form.

2. Building Blocks for a Canopy Reflectance Model

In this section, we develop the ‘building blocks’ that we will require to describe scattering from a vegetation canopy using RT theory. This involves (i) a description of canopy architecture; (ii) a description of the scattering properties of vegetation as a function of wavelength (and polarisation in some cases); (iii) a description of scattering by an underlying (e.g. soil) surface. In the presentation here, we make use of terms normally used for describing optical vegetation canopy radiative transfer.

2.1 Description of Canopy Architecture

For the present, we consider for simplicity a canopy composed of only leaf elements. In canopies comprising e.g. a mixture of leaf and branch elements, the following can be generalised by averaging effects over proportionate distributions of these. Under the assumptions given above, we can describe the canopy using only two terms (Myneni et al., 1989; p. 8):

1. the vertical leaf area density function, \( u_r(z) \) (m\(^2\)/m\(^3\))
2. the leaf normal orientation distribution function, \( g_l(z, \Omega_l) \) (dimensionless).
3. leaf size distribution, defined as area to relate leaf area density to leaf number density, as well as thickness.

All terms can be defined as a function of depth from the top of the canopy \( z \) (or more generally in the 3D case, spatially with the canopy as a function of location \( r \)).

2.1.1 Leaf Area Density

\( u_r(z) \), the leaf area density function, describes the vertical profile of one-sided leaf area density (m\(^2\) of leaf area per m\(^3\) of volume). This term is typically more convenient for optical vegetation applications, where we are typically concerned with projection of leaf areas as the scatterers are large compared to the wavelength under consideration. For a constant leaf area, \( A_l \), it is related to \( N_r(z) \) by:

\[
u_r(z) = N_r(z)A_l
\] (2.1)
where $N_v(z)$ is the leaf number density (number of scatterers per unit volume). A number density is a more convenient description for dealing with extinction and scattering by individual ‘particles’, as in atmospheric radiative transfer or when dealing with vegetation at microwave wavelengths. In optical modelling for vegetation canopies we tend to deal with leaf area density.

The integral of $u_f(z)$ over the vertical extent of the canopy ($H$) is known as the leaf area index (LAI), $L$ (unitless) – one sided leaf area ($m^2$) per unit of ground area ($m^2$):

$$L = \int_{z=0}^{z=H} u_f(z) dz$$

(2.2)

![Figure 2.1 Turbid Medium Representation of Canopy](image)

There are many models of $u_f(z)$ (or $N_v(z)$) we could apply. Many canopies tend to have a higher leaf area density towards the top of the canopy (Myneni et al., 1989). This can either be modelled as a continuous function or by considering scattering between canopy layers.

The simplest form of canopy scattering model is obtained by assuming that density is constant over canopy height ($u_f = L/H$).
2.1.2 Leaf Angle Distribution

The leaf normal orientation distribution function (‘leaf angle distribution’) is defined as a function of the leaf normal vector $\mathbf{\Omega}_i$. It is defined so that its integral over the upper hemisphere $1 (2\pi + \text{sr})$ is unity:

$$\int_{2\pi} g_i(\mathbf{\Omega}_i) d\mathbf{\Omega}_i = 1 \quad \text{(2.3)}$$

Myneni et al. (1989; pp. 13-23) describe a range of methods for the measurement of leaf angle distribution. Many models of leaf angle distribution can be applied. It is most typical to assume that the leaf normal azimuthal distribution is independent of the leaf zenith angle distribution (inclination to the local vertical), i.e.,

$$g_i(\Omega_i) = g_i(\theta_i)h_i(\varphi_i), \left(1/2\pi\right) \int h_i(\varphi_i) d\varphi_i = 1.$$ Most often, the azimuthal distribution is assumed to be uniform ($h_i(\varphi_i) = 1$). This allows for a simpler description of the inclination function of $g_i(\Omega_i)$ as $g_i(\theta_i)$, $\int_{\theta_i=0}^{\pi/2} g_i(\theta_i) \sin \theta_i d\theta_i = 1$. Strebel et al. (1985) and Otterman (1990) suggest however that this may often not be a valid assumption, citing significant diurnal variations in the azimuthal dependance for cotton and soybean crops (Kimes and Kirchner, 1982), as well as intrinsic azimuthal variations seen in tree species such as balsam fir and other factors such as wind, stress effects and heliotropism (Ross, 1981). Goel and Strebel (1984) suggest that for some

\textsuperscript{1} See appendix 1.1 for integration over a hemisphere.
canopies (e.g. soybean) the assumption of independent azimuthal and inclination angles may be valid, although for others (e.g. fir tree needles) it is not.

Typically, we tend to use either simple parameterised distributions or ‘archetype’ distributions (figure 2.3, after Ross, 1981):

- **planophile** — \( g_i(\theta_i) = 3 \cos^2 \theta_i \)
  Leaf normals mainly vertical \( (\theta_i = 0) \);
- **erectophile** — \( g_i(\theta_i) = \left( \frac{3}{2} \right) \sin^2 \theta_i \)
  Leaf normals mainly horizontal \( (\theta_i = \pi / 2) \);
- **spherical** — \( g_i(\theta_i) = 1 \)
  Leaf normals as if ‘pasted’ over a sphere;
- **plagiophile** — \( g_i(\theta_i) = \left( \frac{15}{8} \right) \sin^2 2\theta_i \)
  Leaf normals mainly at \( \theta_i = \pi / 4 \);
- **extremophile** — \( g_i(\theta_i) = \left( \frac{15}{7} \right) \cos^2 2\theta_i \)
  Leaf normals mainly at extremes \( (\theta_i = 0, \pi / 2) \);

\[ \begin{align*}
\sin 2\theta &= 2 \cos \theta \sin \theta \\
\cos 2\theta &= 1 - 2\sin^2 \theta = 2\cos^2 \theta - 1
\end{align*} \]
Several of these have simple mathematical forms and mathematically convenient solutions for average projection and (scalar) scattering functions.

A typical parameterised model distribution is the elliptical function (Nilson and Kuusk, 1989), which provides a convenient and flexible representation:

\[ g_\ell(\theta_l) = \frac{\beta}{\left[1 - \varepsilon^2 \sin^2(\theta_l + \theta_m)\right]} \]

where \( \beta \) is a scaling factor (see equation 2.2). This distribution is defined through the two parameters:

\( \varepsilon \) — eccentricity of distribution : \( 0 \leq \varepsilon \leq 1 \)

\( \theta_m \) — modal leaf angle : \( 0 \leq \theta_m \leq \frac{\pi}{2} \)

The distribution can be visualised as the equivalent of ‘pasting’ the leaves over an ellipse of eccentricity \( \varepsilon \) (zero eccentricity is a spherical leaf angle distribution — \( g_\ell(\theta_l) = 1 \); and eccentricity of 1 is a ‘needle’ — an erectophile distribution for \( \theta_m = 0 \).
and planophile for \( \theta_m = \pi / 2 \), inclined so that most common angle is described by \( \theta_m \). Note that the sensitivity of the function to \( \theta_m \) increases with increasing \( \epsilon \) — for \( \epsilon = 0 \), the function is independent of \( \theta_m \), for \( \epsilon = 1 \), we essentially have all leaves inclined at a single angle \( (\theta_m)^4 \). Because of this sensitivity, eccentricity is often used in a near-linearised sense as \( \ln(\epsilon) \).

Other typical parameterised models include the trigonometric model (Bunnik, 1978) and Goudriaan’s model (Goudriaan, 1977). Note that although we may generally call a particular parameterised distribution by its closest archetype name (e.g. figure 2.4) these distributions are not generally exactly equivalent.

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4 Imagine the effect on angular distribution of rotating a sphere as opposed to rotating a ‘needle’.
2.1.3 Leaf Dimensions

Without modification, RT theory assumes the scatterers to be of infinitesimal dimensions (point scatterers), thus no definition of leaf size is formally required. However, at microwave wavelengths, scattering by objects depends significantly on the dimensions of the leaves, so we must further define leaf size. Simple analytical equations for scattering in the microwave domain are only available for simple geometric primitives (e.g., cylinders, needles, discs) (Fung, 1994; pp 451-473), so leaf dimensions need to be generalised into equivalents for these forms.

At optical wavelengths, scatterer dimensions can also significantly impact canopy reflectance in a certain viewing/illumination configuration (the ‘hot spot’ effect, see below) which can be accounted for by modification of RT theory. In this case, it is not absolute leaf dimension that is important, but rather leaf size (typically a disc radius) relative to canopy height (Nilson and Kuusk, 1989). At optical wavelengths, we may therefore include a relative leaf size parameter in the formulation. Leaf thickness can also be of importance at optical wavelengths, although this is most typically modelled as part of the leaf reflectance and transmittance function. Leaf thickness, often defined as a leaf complexity term (Jacquemoud and Baret 1990) essentially affects the ratio of diffuse reflectance to transmittance of the leaf, a thick/complex leaf having reduced transmittance.

We can generally include a distribution of (absolute, but generalised) element sizes, although this is rarely done for optical models.

2.2 Canopy element and soil spectral properties

In building a canopy scattering model, we consider leaves (and branches) to be the main canopy ‘primitives’ and, in these notes, apply the radiative transfer equation to describing scattering from an ensemble of primitives. This section therefore describes the spectral and polarisation behaviour of vegetation. Since a canopy is typically (lower) bounded by a soil medium, this section also deals with spectral and structural effects of soil scattering.

2.2.1 Scattering properties of leaves

Leaf scattering properties are largely determined by: Leaf surface properties and internal structure; leaf biochemistry; leaf size (essentially thickness, for a given LAI).
• **Leaf surface properties and internal structure**

![Figure 2.5 Dicotyledon leaf structure](http://www.sigu7.jussieu.fr/Led/LED_leafmod_e.htm)

*Scattering* at optical wavelengths occurs within a leaf occurs at air-cell interfaces within the leaf – at refractive index differences which have an overall effect of diffusing the incoming radiation. The refractive index is seen to vary slowly with wavelength, decreasing essentially linearly from around 1.5 at 400 nm to around 1.3 at 2500 nm (Jacquemoud *et al*., 1996). The amount of scattering is seen to depend on the total area of cell wall-air interfaces (Myneni *et al*., 1989; p.33). This can be considered to related to the complexity of the internal structure of a leaf (Jacquemoud and Baret, 1990): the more complex the structure (or the thicker the leaf) the more scattering per unit area, the more diffuse the radiation field (in general), and the lower the transmittance.

In addition, scattering occurs at the surface of a leaf. For optically smooth waxy leaves, a strong specular peak is noted in reflectance. As the leaf surface ‘roughness’ (due to hairs, spines etc.) increases, the specular peak becomes broader (more diffuse) (see e.g. graphs in Myneni *et al*., 1989; p.37).

• **leaf biochemistry**

At optical wavelengths, *absorption* occurs within leaves at specific wavelengths due to pigments water and other leaf materials. The major features are shown in figure 2.6, with major specific absorption coefficients in figure 2.6.

The main pigments are chlorophyll a and b, α-carotene, and xanthophyll (Myneni *et al*., 1989; p. 31), which absorb in the blue part of the spectrum (around 445 nm). Chlorophyll also absorbs radiation in the red, around 645 nm. Radiation absorbed by pigments may be converted into heat energy, fluorescence or converted into carbohydrates through photosynthesis. Leaf water is a major constituent of leaf fresh weight, representing around 66% averaged over a large number of leaf types (Jacquemoud *et al*., 1996). Due to its importance, water is considered separately below. The remainder of leaf weight is ‘dry matter’ mainly composed of cellulose, lignin, protein, starch and minerals. Absorptance by these constituents increases with increasing concentration, thereby reducing leaf reflectance and transmittance at these wavelengths.
Various models of the dependence of leaf reflectance, transmittance (and their sum, single scattering albedo) exist\(^5\), though they typically model only a hemispherical integral of reflectance/transmittance. Nilson and Kuusk (1989) model scattering from the leaf surface by a Fresnel term, simulating roughness effects with a facet model. A common model for flowering plants is the PROSPECT model of Jacquemoud and Baret (1990) which models leaf chlorophyll and water absorption effects in a leaf idealised to \(N\) elemental ‘layers’ using a ‘plate model’ (figure 2.8). In this approach, each ‘layer’ (the solution can be generalised to non-integer numbers of layers) is characterised by a discontinuity refractive index and an absorption coefficient defined by the leaf biochemical composition. Jacquemoud et al. (1996) further developed this model to include other leaf constituents such as lignin and cellulose, and demonstrate how the model can be used to estimate biochemical composition from measured leaf spectra. A similar model, developed to simulate scattering by needle leaves is LIBERTY (Dawson, et al., 1997)\(^6\).

![Figure 2.6 Main regions and factors of leaf absorption](image)

\(^5\) See [http://www.sigu7.jussieu.fr/Led/LED_leafmod_e.htm](http://www.sigu7.jussieu.fr/Led/LED_leafmod_e.htm)

\(^6\) See [http://www.eci.ox.ac.uk/staff/liberty.html](http://www.eci.ox.ac.uk/staff/liberty.html)
Figure 2.7 absorption spectrum for:
(a) chlorophyll; (b) cellulose + lignin; (c) protein; (d) water
(from PROSPECT-redux model: Jacquemoud et al., 1996)

- The leaf water absorption spectrum is shown in figure 2.8(d). It has major absorption features around 1.45 µm, 1.95 µm, and 2.5 µm, but close to zero absorptance in the visible or near infrared part of the spectrum, although there are weak features between .94 µm and .98 µm. As leaf water absorption features are close to soil and atmospheric water absorption, it is not generally straightforward to separate these components, though this may be possible using hyperspectral data. Within the PROSPECT model noted above (Jacquemoud et al., 1996), leaf water content is parameterised as an equivalent water thickness (EWT), which approximates the water mass per unit leaf area. This is readily related to volumetric moisture content\(^7\) (VMC, \(M_v\)) (proportionate volume of water in the leaf) by multiplying EWT by the product of leaf thickness and water density.

\(^7\) [http://www.sowacs.com/feature/IMKO/211_def.htm](http://www.sowacs.com/feature/IMKO/211_def.htm)
leaf dimensions
At optical wavelengths, the general relationship to leaf dimensions is weak at the leaf level as leaf size is always large compared to wavelength (leaf linear dimensions does, however, have an effect at the canopy level as described later). Of course, increasing leaf size increases the total leaf area for a given leaf number density, but since we typically parameterise the canopy through a normalised leaf area term such as leaf area index, this is not a direct effect. We can however restress the effect of leaf thickness noted above on decreasing leaf transmittance relative to reflectance, as, for example described through the leaf structure parameter of the PROSPECT model (Jacquemoud and Baret, 1990).

2.3 Scattering from soil surfaces
For all but the most optically thick canopies, for which the canopy medium may be considered semi-infinite, we require a definition of scattering by a lower boundary surface. Although this may be, for example, snow covered, we consider the more general case of a lower bounding soil surface here.

The main soil properties governing soil scattering are similar for both the optical and microwave case, namely: soil moisture content, soil type/texture, and soil surface roughness.

2.3.1 Soil moisture
At optical wavelengths, the general effect of increasing soil (near surface) moisture is to decrease soil reflectance. The effect is similar in proportion across the optical spectrum, being enhanced only in the water absorption bands.
2.3.2 Soil type/texture
The soil type or texture essentially controls the ‘spectral’ behaviour of soils at optical wavelengths. Perhaps surprisingly, there is generally very little variation in soil spectra: Price (1990) performed a principle components analysis on more than 500 soils from different regions and found that only four spectral basis functions were required to describe 99.6% of the variation in the soil spectra. Stoner and Baumgardner (1982) classified around 500 soil samples from the USA and Brazil and found only five mineral soil spectral curve types (organic dominated, minimally altered, iron affected, organic dominated and iron dominated). These and other results suggest that only a simple parameterisation of soil spectral reflectance (single scattering albedo) is required for canopy reflectance models at optical wavelengths. The basis functions of Price (1990) are well-suited to this purpose.

2.3.3 Soil roughness effects
For a smooth (relative to the wavelength of the radiation) soil surface, lower boundary scattering is simply a Fresnel specular term. This can be important in treating multiple interactions in the microwave case, particularly in forest canopies where the specular scattered field can interact with vertical tree trunks to produce an enhanced backscatter effect. For smooth surface at optical wavelengths, the effect will not generally be significant, except very close to the viewing geometries in the specular direction.

When we are not concerned with polarisation effects (e.g., typically at optical wavelengths), we can simply use the Fresnel power reflectance (Shepard et al., 1993):
\[ \rho = \frac{1}{2} \left[ \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \tan^2(\theta_1 - \theta_2) + \frac{\tan^2(\theta_1 + \theta_2)}{\sin^2(\theta_1 - \theta_2)} \right] \]

In many canopy models, a very simple soil scattering model is all that is required. Typical examples at optical wavelengths are either to simply assume a Lambertian (perfectly diffuse) lower boundary, or to use a simple empirical model such as the modified Walthall model used by Nilson and Kuusk (1989), parameterised by fitting it to soils with a range of roughnesses.

For low (but not zero) roughness, specular effects can be incorporated at by using a distribution of facet orientations and performing an integral over the distribution. At optical wavelengths, increasing RMS roughness increases the angular width of the specular reflectance peak. The same effect is generally true at microwave wavelengths.

For rougher surfaces, such as where there are surface clods or rough ploughing effects, we can use geometric optics models at optical wavelengths for surface scattering. Here, the main phenomenon is protrusion projection and shadow-casting effects. There are few models of soil reflectance which deal explicitly with anisotropic roughness effects. One study which does is the numerical model of Cooper and Smith (1985) which calculates scattering from an arbitrary geometry surface defined using rectangular or triangular facets over an explicitly-defined height field. More recent numerical models such as that of Lewis (1999) can also simulate scattering from such surfaces in investigating soil or topographic effects. Various simpler optical surface scattering models exist, such as the ‘clod’ macrostructure model of Cierniewski (1987) that models roughness (projection and shadowing) effects for a distribution of spheres or other simple forms on a planar surface. In this model, roughness is parameterised as the ratio of the projected area of a sphere to the square of the sphere spacing and the cosine of the slope angle.

Volumetric scattering from soils is generally treated through modifications of radiative transfer theory at optical wavelengths. Hapke (1981) developed a set of models for scattering from particulate soil-like (e.g. lunar) surfaces using such an approach. The model is modified in Hapke (1984) to deal with multiple scales of surface roughness and in Hapke (1986) to deal more explicitly with the soil extinction coefficient and the so-called ‘hot spot’ effect. This latter term can be important for rough soil surfaces, as it can be considered to arise from a decrease in shadow hiding in and around the retro-reflection (‘backscatter’) direction. This results in a peak in reflectance in this region, which increases in angular width with increasing roughness. The half width is shown by Hapke (1986) to be equal to the ratio of average particle radius to extinction length at unit slant path optical depth: effectively a normalised roughness measure. This effect is clearly seen in geometric optics approaches that explicitly consider shadowing.

There are various approaches to modelling surface ‘roughness’ scattering and volumetric scattering terms at microwave wavelengths. As with optical wavelengths, the rougher the soil surface, the more diffuse the surface scattering. Examples of
approaches are the Kirchhoff approximation and the small perturbation model (see chapter 4 of Ulaby and Elachi (1990) and chapter 2 of Fung (1994)).

References

- Sobolev, V. V. 1975, Light Scattering in Atmospheres (Oxford: Pergamon)

See also:
http://www.geog.ucl.ac.uk/~plewis/phd/phdrefs.pdf
http://www.geog.ucl.ac.uk/~plewis/phd/phd3.pdf
http://www.geog.ucl.ac.uk/~mdisney/leaf.html
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