3.03 Canopy Radiative Transfer Modeling

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3.03.1 Introduction

Radiation models are needed to describe quantitatively the absorption and partitioning of solar energy by ecosystems and their components in assessing biosphere–atmosphere interaction. The models are needed also for the interpretation of remotely sensed spectral signatures over vegetated territories. Radiative transfer (RT) in gaseous media (atmosphere) is described by the equation of RT. In the analysis of radiation transfer in the atmosphere, Shifrin (1953) applied the RT concept to the underlying vegetated ground surface as well, using empirical extinction coefficients for the canopy. Quantitative description of the interaction of solar radiation and vegetation is needed in the studies of productivity of vegetation. Studies by Allen, Idso, Monteith, Monsi, Saeki, Warren Wilson et al. in the second half of the past century introduced quantitative description of radiation processes in vegetation canopies. Detailed analysis of RT in vegetation canopies was carried out by the school of Prof. Juhan Ross from Tartu Observatory, Estonia. Ross and Nilson (1963) formulated the RT equation for vegetation. Vegetation canopy was modeled as a plate medium. Extensive studies of vegetation structure and attempts to link measurable canopy structure parameters to the coefficients in the integro-differential equation of RT are summarized by Ross (1981).

The topic turned more actual with emerging satellite remote sensing, and several teams around the world developed theoretical models of RT in vegetation canopies and canopy reflectance (CR) models which are needed in the studies of energy budget of vegetated ground surface and for the interpretation of remote sensing data. The CR problem is to compute the light leaving the canopy in any desired direction. Other quantities of interest are the albedo and fraction of absorbed radiation. Goel (1988) compiled a review of models where he distinguished (a) geometrical models, (b) turbid medium models for homogeneous canopies, (c) hybrid models for heterogeneous canopies, and (d) computer simulation models. The infinite variability of vegetation structure requires simplifications and statistical methods in the description of the structure of vegetation canopy and provides...
possibilities to use different scales and different levels of details in RT models. Not all the developed models fit in with the classification of Goel—there are several mixed varieties of RT models. Validation of RT models in vegetation canopies requires along with radiation measurements—time-consuming measurements of canopy structure. Different models use different levels of abstraction and different sets of input data for describing RT in vegetation. It is not a simple task to justify simplifications and to validate applied approaches. No practical means exist to completely validate one or another RT model by means of optical radiation and canopy structure measurements. In comparable situations different models should give comparable results. To validate is it so or not, comparisons of RT models were launched at Joint Research Center, Ispra (JRC).

### 3.03.2 Radiation Transfer Model Intercomparison (RAMI)

Radiation transfer model intercomparison has been an initiative of JRC to provide a mechanism to benchmark RT models which describe RT at or near the Earth’s terrestrial surface in vegetation canopies and over soil surfaces. Four phases of Radiation transfer Model Intercomparison (RAMI) were carried out in 1999–2010. Description of RAMI is at the Web site http://rami-benchmark.jrc.ec.europa.eu/. Summarizing papers of RAMI results were published in JGR-Atmospheres and Remote Sensing of Environment (Pinty et al., 2001, 2004; Widlowski et al., 2007, 2013, 2015).

In the first phase of RAMI in 1999 the prime objective was to document the variability of CR models when run under well-controlled experimental conditions (Pinty et al., 2001). Three 1D and five 3D models simulated bidirectional reflectance functions (BRF) and hemispherical fluxes in the red and near-infrared (NIR) spectral regions for homogeneous vegetation layer and a heterogeneous layer where foliage was inside spherical envelopes.

In the second phase (2002) the number of participating 3D models increased to 10, and the series of new scenarios including topography and multiresolution BRF simulations were added (Pinty et al., 2004).

In the third phase (2005) the number of participating models further increased. New experiments required calculation of horizontal flux and canopy transmission transect simulations. The geometry of heterogeneous canopies simulated forest stands. Agreement of simulation results between participating models was increased if compared to previous RAMI phases (Widlowski et al., 2007).

In the fourth phase (2009) a completely new set of architectural scenarios was provided. Along with abstract simulated canopies actual canopies were reconstructed from detailed inventories of both the structural and spectral properties of existing plantations and forest stands (Widlowski et al., 2013, 2015).

In following sections the specific features of RT in vegetation are analyzed. In the overview of RT models the main attention is paid to the models which took part in the RAMI exercises. RAMI provided two groups of scenarios for the comparison of models—homogeneous 1D canopy and heterogeneous 3D canopy. Some models could run both groups of scenarios; most of CR models took part in only one group.

### 3.03.3 Equation of Radiative Transfer

RT in a gaseous medium is described by transfer equation. The same formalism can be applied for the transfer of neutrons in nuclear reactors. The theory of RT was developed by Chandrasekhar, Sobolev, Davison, Marchuk et al. For vegetation canopies, one can find detailed mathematical description of the transfer equation and review of publications in works by Myneni et al. (1989) and Knyazikhin and Marshak (1991). Equation of RT describes the local rule of energy conservation averaged over the time period of several periods of the oscillation of electric and magnetic vectors of the electromagnetic field which carries radiation energy. Equation of RT is the equation of energy budget in an elementary volume $dV$. The general form of RT equation is

$$\frac{dl}{ds} = -\sigma(P) P + Q(P, \bar{r})$$  \hspace{1cm} (1)

where $l = I_s(P, \bar{r})$ is the wavelength-dependent radiance at location $P=(x, y, z)$ in direction $\bar{r} = (r_x, r_y, r_z)$ or $\bar{r} = (r, \theta, \phi)$ in polar coordinates, $ds = (dx, dy, dz)$ is the path length in the volume $dV$, $\sigma(P)$ is the extinction coefficient, and $Q(P, \bar{r})$ is the source function. The change of radiance $I_s(P, \bar{r})$ in the path $ds$ is caused by extinction—the first component at the right side of Eq. (1), and adding photons by the source term $Q(P, \bar{r})$. In the case of polarized radiation the balance equation (1) is applied to every Stoke’s component.

Extinction is caused by the absorption and/or scattering of radiation. The source term accounts for the adding of radiation at the path $ds$ which may be caused by the emission of radiation by the volume element $dV$ and by scattering of radiation in the volume element $dV$ to the direction $\bar{r}$ arriving into the volume element $dV$ from any direction $\bar{r'}$. Thermal radiation is absorbed and radiated by every nonempty volume element what must be accounted for in the source function. In optical domain in passive remote sensing the only source is the scattering of solar radiation in the volume element $dV$ which is arriving from the direction of the sun, scattered in the atmosphere, or scattered previously on vegetation elements or soil, Fig. 1.

The source term $Q(P, \bar{r})$ in Eq. (1) is the integral over all $(4\pi)$ incident directions $\bar{r'}$,

$$Q(P, \bar{r}) = \int_{4\pi} I_s(P, \bar{r'}) \Gamma(\bar{r'}, \bar{r}) d\bar{r'}$$  \hspace{1cm} (2)

where $\Gamma(\bar{r'}, \bar{r})$ is the scattering phase function. Note that normalizing coefficients and units are hidden in Eqs. (1) and (2).
Boundary conditions are attached to the RT equation describing the conditions of incident radiation at the upper boundary of vegetation and radiation reflected from the soil or underlying ground layer of vegetation at the lower boundary.

The subject of RT has been investigated and methods of solving RT equation developed by astrophysicists for stellar atmospheres, by physicists for the Earth atmosphere and for the transport of neutrons in nuclear reactors. Success in this field and availability of methods for solving the RT problem encouraged to apply the developed methods for the description of radiation processes in vegetation. However, there are principal differences in transport of radiation in a gas and vegetation. Structure elements of vegetation are large if compared to gas molecules and radiation wavelength and may be close to each other up to building clumps and conglomerates of smaller elements. In a vegetation canopy there are different scales of structure from cell layers in a leaf or conifer needle to shoot, branch, shrub, tree crown, agricultural field, forest stand. The transport of a narrow beam of radiation in a vegetation canopy has binary nature. In the gaps of vegetation there is no interaction of radiation and phytoelements. When the beam hits a phytoelement (leaf, branch, stem), the radiation partly absorbs and partly scatters into hemisphere (stems and branches) or to all directions of 4π solid angle (leaves). The concept of RT equation can be applied only to the averages over some volume $dV$. The RT problem should be solved inside a single leaf or needle and for the vegetated landscape. Attempts to apply the RT equation at all these structure levels meet the principal problem—how to define the volume element $dV$ at every scale and how to express optical properties of this volume element—the extinction coefficient and scattering function—by the measurable structure parameters—the size and number of canopy elements inside the volume element $dV$, their optical properties, and 3D pattern (Nilson and Ross, 1997).

Another specific feature of RT in vegetation is the so-called hot-spot effect. While in a gas the optical properties (transmission, absorption, scattering) in the paths of incident and reflected/scattered radiation can be considered independent, this may not be the case in a vegetation canopy. Due to the finite size of canopy elements and the 3D pattern of scattering/shade-bearing elements the fate of photons scattered in direction $\hat{r}$ may be dependent on the optical path in the incident direction $\hat{r}'$ outside the elementary volume $dV$.

The models of RT in vegetation canopies vary by the scale and details of structure considered, by the methods the RT is handled, and the nature of the problem. Different approach is needed for the global climate models, for the analysis of energy budget and production in vegetation communities, for the monitoring of canopy state using remote sensing tools. The solution of the RT equation is the main tool in turbid medium models for homogeneous canopies. The specific features of incident radiation induce respective methods of the RT equation solution. Unidirectional solar radiation and 5–10 times weaker diffuse fluxes of sky radiation and multiply scattered radiation are better to be handled separately. Geometrical models operate with probabilities of shadowing and seeing sunlit and shaded components of vegetation. Radiances of different scene components may be estimated empirically or using RT methods and models. Computer simulation models follow traveling of photons in a vegetation canopy (Monte Carlo (MC) models) or use some computer graphics methods to calculate scene radiance. In MC models radiation transport is treated as the flux of photons—particles of no size. The main problem is the description of the canopy structure and the fate of a photon if it hits a phytoelement. The solution of the RT problem is reduced to the counting of photons traveling in the given direction. Computer graphics methods operate with probabilities and view factors to calculate incident radiation on every canopy element and probability to see that element. The hybrid methods combine various approaches dependent on the structure of the canopy and incident radiation field.

### 3.03.4 Homogeneous Models

Homogeneous models analyze RT in a horizontally unlimited canopy layer which is treated as a turbid medium of phytoelements. Elementary volume is sufficiently large to contain large number of phytoelements. A typical model of this group is the Ross–Nilson model of plate medium (Ross, 1981). Vegetation elements are small bi-lambertian plates (leaves) described by the hemispherical reflection and transmission coefficients. Orientation of leaves is azimuthally uniform and described by the leaf angle distribution (LAD). Leaves are small compared to the canopy thickness, and so sparse that we can separate the whole canopy layer into elementary layers with no mutual shade-bearing by leaves inside a layer. Transmittance of canopy (gap fraction) is determined by the total area of leaves (leaf area index, LAI) and LAD and follows the exponential Beer–Lambert law,

$$t(\theta) = \frac{\exp(-G(\theta)) \text{LAI}}{\cos(\theta)} \quad (3)$$

where $t(\theta)$ is the canopy transmittance at the zenith angle $\theta$, LAI is leaf area index, $G(\theta)$ is the Ross–Nilson G-function—the projection of unit leaf area in direction $\theta$. 

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**Fig. 1** Radiative transfer in an elementary volume $dV$, $I' = I_0 + dI$. 

$I'$ is the intensity of incident radiation, $I_0$ is the intensity of direct radiation, $dV$ is the volume element, $\sigma$ is the extinction coefficient, $d$ is the size of canopy elements and the 3D pattern of scattering/shade-bearing elements the...
In order to account for the deviations from random structure, Nilson (1971) introduced a clumping parameter $c(\theta)$,

$$t(\theta) = \exp\left(-\frac{c(\theta)G(\theta)\text{LAI}}{\cos(\theta)}\right)$$

Most authors link clumping parameter $c(\theta)$ and LAI together to the “effective LAI.” It is more rational to link together clumping parameter $c(\theta)$ and the geometry function $G(\theta)$. If we rearrange leaves in a canopy the LAI does not change but transparency of the layer, and consequently the area of shadows changes because the overlapping of leaf projections changes. Especially awkward is to talk about changing (effective) LAI with changing view direction. Ni-Meister et al. (2010) link in their ACTS model together LAI, the geometry function, and the clumping parameter into the effective projected LAI.


Turbid medium models assume that the canopy layer can be divided into independent differential sublayers. In real canopies phytoelements may stretch through the significant part or even through the whole canopy layer. An attempt to account for the correlation in transparency of adjacent layers was done by Kuusk (1995b, 2001) and to consider nonleaf elements by Qin and Xiang (1997). The problem of spatial correlation was analyzed by Kostinski (2001). He demonstrated that in a random medium of spatially correlated obstacles the attenuation of incoherent radiation with depth is slower than exponential. In CR models the spatial correlation is accounted for by clumping parameters in the Beer–Lambert law, sustaining exponential extinction. Using of clumping parameters allows to apply homogeneous CR models to heterogeneous scenes (forests, landscape) as well (Pinty et al., 2006; Kuusk et al., 2015).

The scattering phase function of the volume element $dV$ is determined by the leaf optical properties and LAD. Simple models assume bi-Lambertian leaves, having hemispherical reflectance $R_0$ and transmittance $T_1$. Later models consider the leaf bidirectional reflectance distribution including specular reflection on the leaf cuticular fax (Vanderbilt and Grant, 1985). Spectral signatures of leaves are measured or calculated by a leaf submodel. The PROSPECT model by Jacquemoud and Baret (1990) is the most common one, but other options are possible (Baranoski and Rokne, 1997; Dawson et al., 1998; Bousquet et al., 2005; Baranoski, 2006; Stuckens et al., 2009).

Models differ in how the coefficients of the RT equation are expressed via canopy structure and optical parameters, and which method of solving the RT problem for diffuse fluxes is applied. In the SAIL model by Verhoef (1984) fluxes in the sun and view directions are derived for the case of an arbitrary leaf inclination angle and a random leaf azimuth distribution. Diffuse fluxes are calculated using the two-stream approximation. The Kuusk (1995a) model follows directly the concept of plate medium by Ross and Nilson (Ross, 1981), using two-parameter elliptical LAD, and hot-spot correction, still assuming infinitesimally sized leaves in the multiple scattering of radiation. In the SAIL++ model by Verhoef (2002) and models which use the approach of SAIL++ the method of discrete ordinates (DO) is applied for the calculation of diffuse fluxes. In the AddingSD model by Kallel et al. (2008) the adding method of RT is combined by the DO method. Separation of the single-scattered radiation and applying different approach for analyzing single and multiple scattering of radiation introduces some violation of energy conservation which was corrected in the AddingSD, FDM, and FDM-2 models by Kallel et al. (2008) and Kallel (2010, 2012). The drawback was increase of computational cost of models.

### 3.03.5 Heterogeneous Models

The structure of heterogeneous 3D canopy is more complex than of homogeneous canopy, and various structure scales can be distinguished. Heterogeneous models vary by the complexity and by details considered in RT models, thus the variety of models is large. Key issues in developing RT models for heterogeneous canopies are (a) describing the canopy structure, (b) modeling transparency of the canopy for parallel rays (direct solar radiation)—the gap fraction and hot-spot effect in this structure, (c) modeling the phase function of single scattering at every structure scale, and (d) modeling multiple scattering in such structure (Nilson and Ross, 1997).

#### 3.03.5.1 Ray Tracing and Computer Graphics Models

RAMI-III demonstrated high level of agreement between the participating 3D MC models in simulating RT in abstract heterogeneous vegetation canopies (Widlowski et al., 2007). MC models served as surrogate truth for the validation of other models in RAMI-III. MC ray tracing is possible in forward or reverse manner. In forward ray tracing, sample photon trajectories are traced from illumination source through a sensor. In the reverse procedure the trajectories are traced from the sensor to the illumination sources. The forward procedure is preferred if the light is coming mainly from one source which covers a small solid angle (direct solar irradiance). The reverse procedure is preferred in case of overcast sky and narrowly defined view solid angles. Reverse modeling is more efficient than direct modeling, if the sensor has narrow field of view. Calculation of absorption is in inverse procedure more difficult than in the direct procedure.

The computation of RT in a forward MC procedure is composed of four main steps: (1) generation of rays, (2) localization of the ray-object interaction, (3) determination of the type of interaction and the scattering direction, and (4) performing the virtual measurement. The step (3) is described by the probability of absorption and the scattering phase function. While most of MC
CR models use the forward procedure, the FLIGHT model by North (1996) can be used both in forward and reverse procedures. The Drat model by Lewis (1999) uses reverse modeling.

The generation of a scene where rays travel is one of the main tasks of MC experiments. The objects interacting with the radiation field are represented explicitly as 3D geometric structures. Special tools have been developed to build 3D models of plants and trees. Realistic 3D objects are generated using fractal technique and L-systems (Goel et al., 1991; Boudon et al., 2012), the Botanical Plant Modeling System BPMS by Lewis (1999), or detailed data to describe the structure are collected with terrestrial laser scanning (Preuksakarn et al., 2010) or some other 3D digitizing method (Sinoquet et al., 2005).

Another approach is to describe the whole RT process using probabilities—the position of the next interaction is determined by the scattering direction, described by the phase function, and by the distribution of the free path of photons.

Between these two approaches lie hybrid structures where absorbing and scattering elements are collected into restricted volumes (tree crowns). Leaf area density and branch area density within the crown are spatially uniform. Such structure has a discontinuous canopy in the FLiES model by Kobayashi and Iwabuchi (2008), in the SPRINT model by Goel and Thompson (2000), and Thompson and Goel (1998), and in the FLIGHT model by North (1996).

MC models can be easily applied to homogeneous scenes, and for analyzing RT inside a single leaf as well. Problems of MC modeling are: (1) How well do we know the phase function of every interaction? It is simple to assume lambertian phytellements. Using of some more complex phase function increases the number of photons needed for stable results, especially considering almost unidirectional specular reflection on phytellements. (2) Truncation of the tree of interactions introduces the violation of energy conservation. At the same time, MC methods allow to easily collect information on multiple scattering of radiation. There are many MC ray tracing models to choose from. Pbrt by Pharr and Humphreys (2010) is a ray tracer with path tracing integrator.

Drat by Lewis (1999) and Disney et al. (2000) is a MC ray-tracing model, which is driven by 3D locations and orientations of scattering elements combined with descriptions of radiometric properties of the primitive set.

Frat by Disney et al. (2006) and the Librat model by Lewis (1999) and Disney et al. (2000) are the forward versions of the Drat model.

Raytran by Govaerts and Verstraete (1998) is a forward MC ray-tracing model, which allows the explicit representation of radiation transfer in complex scenes. Due to its modular nature, arbitrarily complex canopy scenes may be entered, and further measurement types may always be added.

Rayspread by Widlowski et al. (2006) is an extension of the Raytran model.

Parcinopy model by Chelle (1997) is a MC forward ray tracing, which deals with 3D scene (toric or not) described by polygons and discs having lambertian or non optical properties.

Computer graphics models and radiosity models calculate view factors of seeing the sun, sky, and other canopy elements at the selected canopy element. The radiance of every visible canopy element is calculated, and the scene radiance is the sum of contributions by visible scene elements. Computer graphics models were developed by Goel et al. (1991), Borel et al. (1991), and Gerstl and Borel (1992); the RGM model by Qin and Gerstl (2000); the RGM2 model by Liu et al. (2007) and Huang et al. (2009); and the Hyemalis model by Helbert et al. (2003). The computational cost of computer graphics models is high, and the models are mostly applied to small scenes. It is necessary to divide large forest area into subscenes. The key issue is how to subdivide the large scene and how to consider interaction of nearby or touching subscenes. True heterogeneous vegetation scenes of infinite complexity cannot yet be rendered with current algorithms.

### 3.0.3.5.2 Analytical and Hybrid Models

Analytical models apply some probabilistic scheme to describe heterogeneity of the canopy. Hybrid models combine analytical, numerical, and geometric optical approaches.

#### 3.0.3.5.2.1 Canopy structure

The 4SAIL2 by Verhoef and Bach (2003) has two vegetation layers with coverage below 1, and non-lambertian soil.

The MAC model by Fernandes et al. (2003) is a multiscale analytical CR model based on the angular gap size distribution. The model deals with an arbitrary number of scales of canopy organization. Canopy elements of each scale are nested within the next coarser scale and have statistical distributions.

The ACTS by Ni-Meister et al. (2010) is an analytical version of the GORT model by Li et al. (1995) for calculating gap probabilities as functions of leaf properties, tree geometry, and density. The modeling of both the vertical profile and horizontal clumping of foliage is involved. The clumping function in the Beer–Lambert law was found by fitting the GORT exact values.

In the PARAS model by Rautiainen and Stenberg (2005) and Stenberg et al. (2016) the canopy structure is described by the recollision probability $p$ and escape probability $(1 - p)$. The recollision probability can be defined at different hierarchical levels of the canopy. The total canopy recollision and escape probabilities are decomposed as

$$ \begin{align*}
  p(\text{canopy}) &= p_{1} + (1 - p_{1})p_{2} + \cdots + (1 - p_{1})\cdots(1 - p_{n-1})p_{n} \\
  1 - p(\text{canopy}) &= (1 - p_{1})(1 - p_{2})\cdots(1 - p_{n})
\end{align*} 
$$

where $p_{j}$ is the recollision probability at the hierarchical level $j$. 

The Row model by Zhao et al. (2010) describes RT in a hedgerow with rectangular cross section, which is porous. Within the hedgerow, leaves of given LAD are randomly distributed.

In geometric optical models of discontinuous canopies the foliage is collected into restricted volumes. Most models use some solids of revolution. Tree crowns are modeled as sphere, ellipsoid, cylinder, cone, cylinder capped with a cone, or paraboloid. Tree stems are modeled as cylinders, conical frustums, or described by some taper function. Tree crowns can either rest on the ground surface or have their bases elevated to some height—the crown base. The pattern of tree horizontal positions varies from random (Poisson pattern) to clumped or strongly regular pattern—rows. The random pattern is attractive for models; however, such simulations are inadequate. Competition for light and nutrients drives the tree pattern to more regular, but some species tend to have aggregated pattern. An example of considering competition and aggregation in forming the tree pattern is the electrostatic model by Gusakov and Fradkin (1990). Geng et al. (2016) analyzed the tree pattern in forest plantations. The human influence gives rise to large exclusion distances among trees. The hypergeometric model which has the tendency toward regular distribution is suitable for describing trees distribution in forest plantations.

In the INFORM model by Atzberger (2000) and Schlerf and Atzberger (2006) tree positions are random. Crown shape is not determined, and the canopy layer is described by the likelihood with which an observer sees a crown and ground—the crown and ground factors.

The 4-Scale model by Chen and Leblanc (1997) and the 5Scale model by Leblanc and Chen (2000) have the spatial distribution of trees within a modeling domain and the macroscale geometry of trees. The double-Poisson distribution is used for describing the tree pattern. Trees are combined in groups and the spatial distribution of the center of a group follows the Poisson process, and the group sizes are determined by the Poisson theory.

Nilson (1991) uses the grouping/regularity parameter to consider the deviation of tree positions from Poisson pattern, which is related to the Fisher's grouping index—the relative variance of the number of trees on a plot of certain size. Such tree pattern has the FRT model by Kuusk and Nilson (2000).

Various schemes of crown shape, size, and position are in use: trees of equal size, of equal size crowns having some distribution of crown height, crowns of different form (e.g., ellipsoid and cone), fixed number of crown sizes, or the continuous distribution of crown size.

In the GORT model and the SGORT model by Ni and Jupp (2000) the canopy layer is modeled as randomly distributed spherical crowns inside a layer of finite thickness. In the CanSPART model by Haverd et al. (2012) tree crowns are modeled as spheroids, and trunks as cones with base diameter equal to $D_{\text{Bre}}$—the breast height diameter. In the 5Scale model, tree crowns have been modeled with a cylindrical shape of given length and radius, and with a conical top of half apex angle $\alpha_c$. In the FRT model tree crowns are ellipsoids or cylinders with a conical top. Tree stems are described by a taper curve. Several classes of the fixed size of trees and tree crowns are possible.

The DART (Discrete Anisotropic Radiative Transfer) model by Castellu-Etchevery et al. (1996) and Castellu-Etchevery et al. (2015) describes a heterogeneous 3D scene by dividing the scene into a rectangular cell matrix. The model does not require that the individual cells that constitute the 3D scene have equal dimensions. The numbers of cells along the vertical and horizontal axes can be independent. Very complex 3D scenes can be modeled—terrain geomorphology, forest stands, agricultural crops. The model can work with scenes that comprise different types of elements (leaves, grass, soil, water, tree trunks, buildings) with any 3D distribution. The model is deterministic when dealing with the shape and spatial distribution of the objects that make up the scene.

Details of the internal structure of tree crowns vary between models. In the CanSPART model and in many other models tree crowns are porous. Several models have spatially uniformly distributed flat leaves in the crown. Leaf orientation is described by the distribution of leaf normal $g_l(\theta, \phi)$. Although some leaf azimuth may be preferred in some canopies/tree crowns, CR models use uniform distribution of leaf azimuth, and leaf inclination is described by the LAD. In models leaves may have spherical distribution of leaf angles, uniform inclination of leaves, fixed leaf angle, or LAD described by some one- or two-parameter distribution function. Campbell (1986) suggested ellipsoidal LAD which allows to model continuous change from planophile to eutrophic LAD changing the ratio of the vertical and horizontal semiaxes of an ellipse. The most frequently used two-parameter LAD is the beta distribution. Unfortunately the relation of the symmetrical parameters of the beta distribution have no obvious link to the measurable parameters of LAD—the mean inclination and the standard deviation of leaf angle. Kuusk (1995a) suggested two-parameter elliptical LAD, where LAD in polar coordinates is a sector of an ellipse. The modal inclination of leaves is the inclination of the longer semiaxis of the ellipse, and the shorter semiaxis determines the width of LAD.

In tree crowns of uniformly distributed foliage the transparency of a crown follows the Beer–Lambert law Eq. (3), determined by the foliage volume density and path length inside the crown. Nontransparent tree branches and stem inside the crown must be accounted for in the calculation of crown transparency. The clumping of leaves and needles into shoots is accounted for by adding a clumping parameter into the transparency equation, Eq. (4). The value of the clumping parameter depends on the tree species, and it has been estimated empirically or by MC simulations. The consideration of more realistic crown structure in CR models—the change of the balance of leaf/needle and branch area as a function of distance from crown perimeter or stem, the aggregating of shoots into branches, the change of leaf/needle area density with height—makes the calculation of the single scattering of direct solar radiation inside a crown very complex. Such details of the crown structure can be considered in MC ray-tracing models.

Gap probability in a discontinuous canopy consists of gaps between individual crowns and of within-crown gaps. The area of shadows cast by tree crowns and stems depends on the tree and crown dimensions, view angle and tree pattern, and on the
transparency of tree crowns, discussed earlier. Along with the gap probability in sun direction the bidirectional gap probability in sun and view directions is needed in the calculation of single scattering of direct solar radiation. The dependence of the bidirectional gap probability on the angle between the sun and view directions is behind the hot-spot effect. In a discontinuous canopy the between trees and inside crown hot-spot effects form the angular pattern of leaving radiance. Nilson (1999) provides expressions for calculating gap probability in a forest canopy at level $z$ in direction $\theta$ for nontransparent tree crowns as a function of the crown projection area in direction $\theta$ and the stand statistical pattern,

$$t(\theta) = \exp(-c(\theta)C_{CR}K(\theta)).$$

(6)

where $C_{CR} = \text{NS}(0)$ is the crown closure above the level $z$, $\text{NS}(0)$ is the crown projection area in the vertical direction, $N$ is the stand density, and $K(\theta)$ describes the relative change of projection area along with a change in the view direction $\theta$. The regularity parameter $c(\theta)$ is determined by the Fisher's grouping index $GI$, $c(\theta) = (\ln (GI))/1 - GI$.

Multiscale light interception models (e.g., MuSLIM model by Da Silva et al. (2012)) for single trees were introduced by Da Silva et al. (2008). By making use of 3D digitized fruit trees, tree architecture was decomposed into different structural levels, such as leaves, current-year shoots, 1-year old shoots, scaffolds, and crown. For each level, a convex envelope is defined while the porosity of the structural unit depends on the foliage area volume density and on the spatial dispersion (actual, regular, random) of the lower-order structural level within the envelope. This way, the importance of all structural levels and of the spatial distribution pattern can be studied. In addition, the structure of individual trees can be linked to the genetically determined topology and geometry and to the light interception efficiency. The latter is typically described by the silhouette to total area ratio (STAR) (Oker-Blom and Smolander, 1988; Duursma et al., 2012).

The bidirectional gap probability needs the calculation of the overlapping area of crown projections in sun and view directions,

$$t(\vec{r}_s, \vec{r}_v) = \exp(S_s + S_v - S_w)$$

(7)

where $S_s$ is the shadow area, and $S_v$ would be the shadow area if we replace the observer by sun, $\vec{r}_s$ and $\vec{r}_v$ are the sun and view directions, respectively. $S_w$ is the overlapping area of the shadows $S_s$ and $S_v$. Overlapping of crown projections depends on the crown shape, sun and view geometry, and tree distribution. The formula for the overlapping area $S_w$ is rather simple in the case of spherical tree crowns, but rather complicated in the case of more complex crown shapes (e.g., cone atop a cylinder and conical stem). The Nilson’s scheme of calculating between-crown gap fraction is applied in the FRT model. The exact formulae of overlapping area $S_w$ are replaced by the approximation where crown projections are replaced by circles of respective area placed at the center of crown projection. The transparency of tree crowns in direction $\theta$ depends on the mean path length inside the crown and volume density of crown elements and is accounted for in the calculation of shadow area.

Other options for the calculation of crown-level bidirectional gap probability are applying the concept of flying spheres by Seeeliger (1887) or of porous surface by Hapke (1963). The Seeeliger’s spherical opaque particles are replaced by tree crowns, and the hot-spot profile is determined by the share of common volume in the total volume of empty cylinders in the forest canopy in the sun and view directions at level $z$. Hapke (1963) modeled pores as cylinders of statistical distribution of diameter and depth. Bidirectional gap probability is the probability that the bottom of the cylinder can be seen from the sun and view direction.

In the SScale model the crown level hot spot is calculated using the gap size distribution on ground surface, which can be measured in a forest,

$$t(\vec{r}_s, \vec{r}_v) = t(\theta)t(\theta_s) + (t(\theta_s) - t(\theta_s) t(\theta_v))F_{hs}(\alpha)$$

(8)

$$F_{hs}(\alpha) = \frac{\int_{L_{min}}^{\infty} F_{hs}(\alpha, \lambda_2)N(\lambda_2)d\lambda_2}{N_g}$$

$$F_{hs}(\alpha, \lambda_2) = 1 - \frac{\alpha}{\tan^{-1}(\lambda_2/H)}$$

Here, $F_{hs}(\alpha, \lambda_2)$ is the hot-spot function for the gap size $\lambda_2$, $\alpha$ is the angle between the sun and view directions, $H$ is a measure of the crown shadow, which depends on the crown shape and sun zenith angle, $N(\lambda_2)$ is the distribution density of gap size, $N_g$ is the total number of gaps, and $l_{min}$ is the minimal size of a gap at which the sun and view directions are open at level $z$. The area of shadows cast by tree crowns is corrected by crown transparency calculated by Beer–Lambert law but corrected for foliage clumping in crowns.

Rather similar approach for the crown level hot spot is in the Semidiscrete model by Cobron et al. (1997). In the DART model the hot-spot concept of a homogeneous layer by Kuusk (1991) is applied at every scale.

In the INFORM model the statistical approach by Rosema et al. (1992) is used. The dependence of shadow areas in the sun and view directions is accounted for by an exponential correlation coefficient $r = \exp(-g_\alpha/d_\alpha)$, where $g$ is a geometrical factor depending on the sun and view geometry, $z_c$ is the height of the crown above the ground, and $d_c$ is the crown diameter. A similar correlation is introduced in the GORT model, but the calculation scheme is different.

### 3.03.5.2.2 Single scattering of radiation

Single scattering of radiation in the view direction combines the scattering of radiation inside canopy and the reflection of uncollided radiation on ground surface. The contribution of the volume element $dV$ inside a tree crown is determined by the bidirectional
gap probability \( t_{\text{fl}} \) and by the scattering phase function of the volume element \( dV \). In the Ross–Nilson plate medium the phase function of the volume element \( dV \) which contains the unit area of plates (leaves) is

\[
\Gamma(\vec{r}_s, \vec{r}_v) = \int_{\pi} r_L(\vec{r}_L) \gamma_L(\alpha) |\cos \alpha| |\cos \gamma| d\vec{r}_L,
\]

(9)

where \( g_L(\vec{r}_L) \) is the 2D distribution of leaf normals \( \vec{r}_L \), \( \alpha_s \) and \( \alpha_v \) are the angles between the leaf normal \( \vec{r}_L \) and sun direction \( \vec{r}_s \), and between the leaf normal \( \vec{r}_L \) and view direction \( \vec{r}_v \), respectively, \( \gamma_L(\alpha) \) is the scattering phase function of a leaf, \( \alpha \) is the angle between \( \vec{r}_s \) and \( \vec{r}_v \).

Most hybrid models use the phase function Eq. (9) inside tree crowns, where phytoelements are bi-lambertian scatterers described by reflectance \( R_L \) and transmittance \( T_L \) and in some models consider the specular reflection on the leaf/needle surface. Reflectance and transmittance of scatterers are averaged over the volume element \( dV \) and consider branches and stem parts as well. The assumption of the phase function Eq. (9) is that the volume element \( dV \) is large enough to contain large number of scattering elements so that their contribution can be described statistically, but, at the same time, there is no mutual shadowing of elements. We cannot find such volume element in real tree crowns. In the FRT model by Kuusk and Nilson (2000) the reflectance \( R_L \) and transmittance \( T_L \) of a single leaf inside the volume element \( dV \) were corrected to compensate for overlapping of leaves in the sun and view directions. The comparison of the BRF simulated with FRT to measurements in the principal plane shows that the model is not able to reproduce accurately the balance of backward and forward scattering of coniferous forests. Therefore, Kuusk et al. (2014) demonstrated that, except the in-crown hot-spot peak, the Henyey–Greenstein phase function models well the balance of backward and forward scattering by a single tree crown. Mõttus et al. (2012) found that a single Scots pine shoot is similar to a single tree crown a backward scatterer, the backscatter of which is not directed into a narrow peak around the hot-spot direction, and the phase function of which can be approximated with Eq. (10). Liang and Strahler (1995) applied the Henyey–Greenstein phase function in the calculation of multiple scattering in vegetation canopy. There is no physical model for estimating the asymmetry parameter \( g \) of Eq. (10) for foliage clusters (shoot, branch, tree crown). In Kuusk et al. (2014) it was estimated by comparing the simulated and measured BRF-s of forest stands. Fig. 2 demonstrates the role of phase function in the forming of BRF of a forest stand. Angular distribution of the red directional reflectance (660 nm) of the Järvselja pine stand in Estonia is compared to simulations with the FRT model. The stand was one of the actual scenarios at RAMI-IV. BRF measurements were done on July 24, 2008 at sun zenith angle SZA = 44° in conditions of clear sky and good transparency of the atmosphere. The share of diffuse sky flux in the total spectral irradiance was 5%, thus the recorded BRF is almost bidirectional reflectance factor. Altogether 13,882 samples of directional reflectance are collected into groups of 2° view zenith angle and 4° azimuth angle in the range of 0–65° zenith angle, 0–10° azimuth angle at the backscattering side, and 0–20° azimuth angle in the forward scattering side. Every dot in Fig. 2 is the average reflectance factor in the group. Group sizes vary from 6 to 199 samples. More details on the measurements and data processing are reported in Kuusk et al. (2014). Parameters of LAD were adjusted by fitting simulated gap fraction angular profile to that on hemispherical photos.

**Fig. 2** BRF of the RAMI pine stand in the solar principal plane (points) compared to simulations with the FRT model: FRT\(_{\text{HG}}\), FRT\(_{\text{T}}\), and FRT\(_{\text{HG}}\)—using Ross–Nilson phase function and spherical or erectophile elliptical LAD, respectively; FRT\(_{\text{HG}}\)—using the Henyey–Greenstein phase function.
Contribution of the single scattering on ground surface in the BRF of a discontinuous canopy is determined by the bidirectional gap fraction $t_g$ on ground surface and by the BRF of ground. If the forest canopy is closed the contribution of ground reflectance to the CR is small and inaccuracies in ground reflectance do not affect the modeled CR much; therefore, several models use the simplest lambertian surface. Other options are to use parabolic approximation to model asymmetry of soil BRF (Walshall et al., 1985) or the Hapke model of porous layer (Hapke, 1963), or some homogeneous vegetation reflectance model (ACRM in the FRT model) as the ground surface.

The quality of reproducing spectral signatures of vegetated surfaces by models depends on the provided soil reflectance spectra, on the pigments considered in leaf models, and on the adequacy of absorption spectra of pigments. All these spectral variables vary in large range and are difficult to measure.

### 3.03.5.2.3 Multiple scattering of radiation

The most common approach to model multiple scattering and the transfer of diffuse sky radiation is to use some simple solution of RT equation. The simplification that multiple scattering is independent of azimuth angle is accepted. This assumption cannot be valid in a forest stand of separate dense tree crowns where shade sides of crowns receive the single-scattered radiation from sunlit crowns.

In the azimuthally uniform case the 2D integral of the source function Eq. (2) reduces to the 1D integral over zenith angle. In the discrete-ordinate method of RT the continuous integral Eq. (2) is replaced by the sum over finite set of prescribed directions. So the integro-differential RT equation Eq. (1) is replaced by the set of differential equations. Special cases of the discrete-ordinate method are the two-stream and four-stream solutions for RT. In the two-stream approximation the RT is described by vertical fluxes up $E_+$ and down $E_-$. Liou (1974) provided explicit analytic solution for the RT in the atmosphere with discrete streams of four, and Liang and Strahler (1995) applied this RT solution for vegetation canopy. Vertical fluxes up $E_+$ and down $E_-$, a direct solar flux $E_s$, and a flux associated with the radiances in the direction of observation $E_o$ are linked by the set of differential equations

\[
\frac{dE_+}{dz} = -auLE_+ + \sigma uLE_+ + s'uLE_s, \\
\frac{dE_-}{dz} = -\sigma uLE_+ + auLE_- - s'uLE_s, \\
\frac{dE_s}{dz} = kuLE_s, \\
\frac{dE_o}{dz} = vuLE_- + wuLE_+ - KuLE_o,
\]

where $u_L$ is the leaf area density, coefficients $a, \sigma, s', s, k, v, u,$ and $K$ are determined by the geometry function $G(\theta)$, leaf reflectance $R_L$, and transmittance $T_L$. In discontinuous canopies the leaf area density $u_L$ is horizontally averaged but may have some vertical profile, $u_L = u_L(z)$. The options are to approximate the whole canopy layer by a homogeneous layer in the two-stream (four-stream) problem or separate canopy into horizontally homogeneous stacked layers, and the two-stream or four-stream solution is found separately for every layer. For the concatenated canopy layers the adding method can be used (Kuusk, 2001; Kallel, 2012). To specify a unique solution of Eqs. (11), it is necessary to specify the incident radiation intensity at the canopy boundaries, that is, at ground surface and at the top of canopy in case of a single layer or at the upper and lower boundaries in case of sublayers.

The two-stream scheme is in use in the ACTS and CanSPART models, and a four-stream scheme in the SAIL, FRT, and Row models. In the two-stream approximation diffuse fluxes are considered isotropic. Four-stream approximations allow to model the directional variations of diffuse fluxes. The DART and Rayspread models allow to calculate horizontal components of multiple scattering between scene cells. In the GORT model by Ni et al. (1999) discontinuous canopy is divided into a homogeneous vegetation layer where tree crowns are squeezed together and a layer without vegetation. The RT problem of diffuse fluxes is solved separately in these two parts of canopy, and total fluxes are the sum of these component fluxes, weighted by areal proportions of respective components.

Successive orders of scattering are considered in MC models, in the DART and Semidiscrete models. These models use also the discrete ordinates technique for modeling RT. The adding method of RT is used in the FDM model.

In the radiosity models sunlit leaves are illuminated by the direct sunlight, diffuse skylight from canopy gaps above the given leaf, and the diffuse light scattered from other canopy elements. Shaded leaves are illuminated only by skylight and multiply scattered light. View factors allow to account for the azimuthal anisotropy of multiple scattered radiation. An attempt to apply view factors for multiple scattering in a geometric-optical model was done by Chen and Leblanc (2001).

### 3.03.5.2.4 Radiative transfer inside a leaf

In CR models where leaf optical parameters are not the given input parameters, the most popular leaf model is the PROSPECT model by Jacquemoud and Baret (1990). Leaf is modeled as a pile of $N$ homogeneous layers separated by air spaces. The two-stream model of propagation of diffuse radiation in diffuse scattering media by Kubelka and Munk (1931) is applied. The hemispherical reflectance $R_1$ and transmittance $T_1$ of such leaf are determined by the refractive index of the leaf material, incidence angle $\theta_i$ of illuminating radiation, and absorption coefficient of the leaf material. The incidence angle $\theta_i$ is fixed ($\theta_i = 60^\circ$), and refractive index is tabulated as a function of wavelength in the model code. Leaf biochemistry components in the first version of PROSPECT...
were water, chlorophyll \((a + b)\), and some albino leaf, described by the tabulated absorption spectra. In the RT calculations a single effective absorption coefficient \(a(\lambda)\) is used which is the weighted sum of component absorption coefficients. Next versions of PROSPECT vary by the number of biochemistry components considered. In the ACRM and FRT models the variant of the PROSPECT is used where the absorption spectra of leaf pigments are tabulated in separate files, and any number of pigments can be used. The selection of pigment spectra controls the spectral signature of the leaf. Varying the structure parameter \(N\) the balance of leaf reflectance, transmittance, and absorption are adjusted. In these two CR models (ACRM and FRT) the leaf structure parameter \(N\) (the number of cell layers in a leaf) is corrected with the clumping parameter \(c(\theta)\), \(N_{\text{eff}} = N/c(\theta)\) in order to compensate the overlapping of leaves in a clumped canopy—overlapped leaves are treated as a thicker leaf of \(N_{\text{eff}}\) cell layers.

The plate model does not fit well for conifer needles. Dawson et al. (1998) suggested the LIBERTY model which is based on the Melamed (1963) theory of light interaction with suspended powders. A conifer needle is modeled as a volume of aggregated spherical cells of given diameter which are separated by air gaps. Similar to the PROSPECT model, the spectral signature of a needle is determined by the absorption spectra of pigments. The input parameters of the model are, along with pigment absorption spectra and concentrations, the cell diameter and the size of air space between cells. The LIBERTY model is an option for foliage optics in some CR models (5Scale, INFORM, FRT, ACRM).

In the DLM model by Stuckens et al. (2009) the impact of leaf asymmetry on RT is modeled. The plate model of PROSPECT is generalized for conditions where light inside a layer is not necessarily fully diffused. A leaf is modeled as a stack of four nonidentical layers. Adjacent layers are separated by air spaces for only a fraction of their surface area. The reflectance and transmittance of an entire leaf consists of a combination of reflectances and transmittances of sublayers. Leaf asymmetry is modeled by assigning nonuniform distributions of pigments, water, and dry matter to sublayers.

### 3.03.6 Concluding Remarks

The infinite variability of vegetation structure complicates the modeling of RT in vegetation canopies. Numerous models of RT in vegetation canopies were developed in the second half of the last century. Models differ by the details accounted for and by the simplifications introduced in the description of canopy structure and photon–vegetation interactions. No practical means exist to completely validate one or another RT model by means of optical radiation and canopy structure measurements. In order to document the variability of developed CR models, to validate models, and to help to orientate among the developed models inter-comparisons were started at JRC as the RAMI initiative. While the first phase of RAMI in 1999 was introductory where the specifications of simulation experiments were developed and tested by few CR models, the second phase (RAMI-II) in 2002 had the expanded set of test cases and participating models. All test cases of RAMI-I were run in direct mode again. The models that have been upgraded were reevaluated, and new models were accommodated. New test cases included significantly heterogeneous environments for the performance tests of 3D models. Agreement between participating models was best in simulating NIR reflectance of a homogeneous vegetation. Discrepancies between models of simulating red reflectance were 3%–5%. The agreement of models in simulating RT in heterogeneous environments was much worse. The collective behavior of a group of MC 3D models was indistinguishable from a RT point of view over particular sets of tests. The scattering of simulation results by analytical and geometric-optical models was 10% and more.

Both the number of test cases and participating models increased in the third phase of RAMI in 2005. Models demonstrated increased conformity. Six MC models (DART, Drat, FLIGHT, Rayspread, Raytran, and SPRINT) were in very close agreement with each other, and their results were used as surrogate truth for the validation of other models. Fifteen models ran homogeneous test cases. The model-to-ensemble agreement of results was within 2%–4%, and the only deviator was MBRF. In the heterogeneous case models with structural and radiative approximations deviated somewhat more. Deviations from the ensemble varied from test to test. Rather large deviations in some tests were observed by complex as well as more simple models.

Dispersion of simulation results using actual canopy scenarios in RAMI-IV (2009) was found to be rather large for flux simulations as well as BRFs. Possible causes for such dispersion could be (1) operator choices and the capability of models to adapt test case descriptions to the needs of a given RT model, (2) operator errors during the setup of the actual model run, and (3) model errors—conceptual errors or software bugs. Validation of models toward in situ observations was beyond the strength of RAMI. The approach of previous RAMI phases was applied in RAMI-IV as well, and the quality of simulations was evaluated by the consistency between the generated RT quantities, deviations from an absolute truth criteria where available, and proximity to a reference (surrogate truth).

One important test was the test of energy conservation by models. There are possibilities to violate energy conservation by the choice of methods and simplifications in the analysis of RT. The separation of single scattering and multiple scattering and application of different principles for the description of the first scattering and higher-order scatterings unbalance the radiation energy accounted for in the module of the single scattering of models, and radiation energy of first scattering, dismissed in the module of multiple scattering. These differences are caused, as a rule, by considering the hot-spot effect, by using different phase function for the single and multiple scattering of radiation in the canopy, and by using different BRF of ground for uncollided radiation and multiple scattered radiation. The errors, caused by this disbalance, may be acceptable in wavelength ranges of high absorption when the contribution of multiple scattering is small. Other schemes of possible violations of energy conservation are the small number of scattering orders accounted for in the method of successive orders of scattering in solving RT equation or in MC procedures, and problems in selecting scattering directions in the discrete ordinates method of RT. The CanSPART model showed a perfect
energy balance record, FLiES exhibited minimal deviations, but Rayspread and RGM models showed deviations up to 2% in NIR. Surprisingly high was disbalance of the DART results varying between 5% and 40% by absolute value in some tests.

Success in simulating RT quantities by models varied from test to test at RAMI-IV. In conformity testing successful models were INFORM (BRF in perpendicular plane, transmission of direct radiation), Pbrt (BRF in azimuthal ring, black sky albedo), FRT (transmission of diffuse radiation), FLiES (white sky albedo), Rayspread, RGM, Librat, and Flies (BRF of the citrus orchard).

Model comparisons like RAMI help to reveal problems in models which could not be noticed when using model with a small set of input parameters and check the limits where the assumptions behind models are valid and simplifications in models are acceptable. It is possible that in some cases some model conceptual errors are compensated by other errors. Models should be consistent. The unidirectional and bidirectional gap probabilities, the recollision and escape probabilities of scattered photons, and other structure functions should be spectral invariants.

The summary list of models referred to is in Tables 1 and 2. Selecting the best model for a given application is a delicate task. No single modeling approach is superior in all instances. Models are complex, and preparation of input data and running the models needs assistance of model developers in order to avoid operator errors during the setup and run.

There are some aspects of radiation propagation not considered here. Coherent backscattering may have some role in forming the hot-spot effect. Coherent backscattering is analyzed in works by Hapke (2002), Tishkovets and Mishchenko (2004), and others. The coherent part of the radiation reflected by the medium is caused by the interference of multiply scattered waves in a narrow region of scattering angles in the vicinity of the exact backscattering direction. Possibilities for coherent effects are higher in the media of densely packed particles (regoliths, dust). Vegetation elements and gaps in vegetation are large if compared to radiation wavelength, and the finite angular size of the Sun smooths the field of scattered radiation. Therefore we can hope that ignoring coherent backscattering of radiation is not a big problem in CR models.

The above treatment misses fluorescence as well. Fluorescence redistributes energy between wavelengths and adds sources of radiation inside the canopy in the last term at the right side of the transfer equation Eq. (1). The energy of emitted radiation is small compared to the scattered sunlight (about 1%–5% in the NIR) and should be accounted for only in special studies where fluorescence serves as the indicator of vegetation conditions and biochemistry processes (Guyot, 1993; Verrelst et al., 2016).

Row structure of plantations and sloping terrain makes the modeling of RT even more complex, and as a rule task-specific models are needed. Exposition to the direct sun radiation depends on the slope and row azimuth; exposition to the diffuse sky

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</table>

DO—method of discrete ordinates; GO—geometrical optics.
Table 2  Computer simulation models

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<th>Model</th>
<th>RT formalism</th>
<th>Reference(s)</th>
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<td>DIANA</td>
<td>Radiosity + L</td>
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<td>Drat</td>
<td>Ray-tracing + L</td>
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<td>Radiosity</td>
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<td>RGM2</td>
<td>Radiosity</td>
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<tr>
<td>SPS</td>
<td>Photon spread</td>
<td>Thompson and Goel (1998)</td>
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<td>Leaf models</td>
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<td>ABM-8, ABM-U</td>
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</table>

DO—method of discrete ordinates; GO—geometrical optics; L—L-systems.

radiation depends on the slope inclination. Terrain slopes and row structure introduce the azimuthal anisotropy of multiple scattering. Terrain slopes are explicitly present in 3D ray-tracing models. During RAMI-III, except the ray-tracing models, the topography test case was run only by the MAC model. The scene with regular planting pattern during RAMI-IV also was analyzed with ray-tracing models and the hybrid Row model.

See also: 5.04. Top of Atmosphere Broadband Radiative Fluxes From Geostationary Satellite Observations.

References


**Relevant Website**

RaDiation transfer Model Intercomparison RAMI—http://rami-benchmark.jrc.ec.europa.eu/)