Q1
i) Give a linear differential equation describing the change of a population with time t. You can assume your population is resource-limited i.e. that it cannot exceed a fixed carrying capacity, P. [10 marks]

ii) Derive the general solution of this equation (showing your working) and explain each term in the general solution. Describe how the population varies as a function of any constants in the resulting expression. You should use figures where appropriate. [40 marks]

iii) Given that general solution of $d^2y/dx^2 + y = 0$ is $y = A\cos x + B\sin x$, find particular solution which satisfies the boundary conditions $y = 0$ when $x = 0$ and $y = 3$ when $x = \pi/2$ [50 marks]

A (part iii)
First condition gives $0 = A\cos 0 + B\sin 0 = A$, hence $A = 0$. Second condition gives $3 = B\sin \pi/2 = B$, hence $B = 3$. So required solution is $y = 3\sin x$.

Q2
i) Give examples of two-parameter linear and non-linear models relating an independent variable $y$, to a dependent variable $x$. [30 marks]

ii) Describe ONE advantage and ONE disadvantage of using a linear model, as opposed to a non-linear one. You should use figures where appropriate in your answer. [30 marks].

iii) Describe ONE application of a linear model for parameter estimation. [40 marks]

Q3
i) Describe a numerical method by which you might integrate a function for which an analytical integral is not available. You should make clear the limitations of this method and use figures where appropriate. [30 marks]

ii) Describe TWO methods of non-linear model parameter estimation (inversion), at least one of which should NOT require calculation of function derivatives. You should make clear any assumptions and limitations of the methods you describe. [70 marks]
Q4

i) Describe TWO key advantages and ONE key disadvantage of the use of Bayes Theorem to calculate probability. [30 marks]

ii) You are given a sequence of coin tosses as follows: HTHHHTHTHH i.e. 7H & 3T. Using a flat (uniform) prior probability of 0.5, use Bayes Theorem to calculate the posterior PDF describing the possible range of bias weightings over the interval 0-1, in steps of 0.2. [40 marks]

iii) Describe an alternative prior that could be used in and the effect of it would have on the outcome of your calculation in ii). [30 marks]

Q5

i) Use figures to show 2 examples of time series of observations with different characteristics, and describe how such series might arise in practice. [40 marks]

ii) For EACH of these series, describe in detail the method(s) you would use to model the properties of the time series. You should lay out any algebraic equations required, defining the terms of these expressions. [60 marks]